A LINEARIZED IMPLICIT SCHEME FOR SOLVING LOGICAL NONLINEARITY: THE SCHEDULING EQUATION

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ABSTRACT: As part of this research, the Linearized Implicit Scheme, a numerical approach, was developed. The calculations will take much less time, and the solution to the nonlinear logarithmic Schrodinger's equation will be accurate to the second order in both space and time. The results are then compared to those obtained before using the Crank-Nicolson scheme of the finite difference approach. Before it is used, this procedure will be tested for accuracy and stability. In this study, conserved amounts and accurate solutions are utilized to demonstrate that the proposed method works and can be depended on. In addition, research is being conducted to determine how two and three solitons communicate with one another. According to the data we acquired for our research, interactions are similar to elastic properties.

Keywords: Linearized implicit scheme; exact solutions; stability; bounded domain; one soliton; gaussons; soliton interaction.

1.INTRODUCTION

Nonlinear Schrodinger's (NLS) equation can be applied in a variety of fields, including fluid dynamics, biomathematics, nonlinear optics, and plasma physics. One of the most important areas of nonlinear optical research in recent years has been the study of how solitons move in optical signals. Since the turn of the century, much research has been conducted on optical solitons with log-law nonlinearity, also known as optical Gaussons. A large amount of scientific data has previously been made public. The topic of integrability has piqued the interest of many experts from around the world. The NLS equation does not operate in these nonlinear forms when expressed in this manner. Log-law nonlinearity is a type of nonlinear dynamics. It is more difficult to set error bounds for Schrodinger's (log NLS) equation and come up with novel ways to employ numbers as a result of the logarithmic nonlinearity explosion. Because there are no apparent domain boundaries and the process is not linear, numerically solving Logarithmic Schrodinger's equation across unbounded sectors is extremely difficult.

Biswas and Aceves developed a method for studying visual solitons by perturbing them in 2000. Soliton qualities are derived from a modified nonlinear Schrödinger equation . Biswas investigated how disturbance terms affect the cooling of optical soliton in 2008. In the same year, Kohl et al. investigated a number of nonlinearities, including power law, hyperbolic law, dual-power law, and Kerr law. Biswas et al. discovered a correct one-soliton solution to Schrodinger's equation in 2010, demonstrating log-law nonlinearity and unstable disturbances. Khalique and Biswas devised a method in 2010 to merge log-law nonlinearity and Schrodinger's equation into a single equation. The Lie symmetry approach was used to accomplish this. This enabled tentative responses to be found. Biswas and Milovic discovered a single soliton solution to Schrodinger's equation that is not consistent with the log law. The solitary wave approach was employed to accomplish this.... According to Biswas et al. (2011), Schrodinger's nonlinear equation for optical solitons is implemented by combining non-Kerr law nonlinearity with perturbation components that exhibit full nonlinearity. The goal of this investigation was to see if there were any disturbance elements that demonstrated high nonlinearity . Biswas et al. published a paper on the variational principle-based soliton solution in nonlinear optics in 2012. Alex demonstrated in 2016 that the ground state of the logarithmic Schrodinger's equation remains orbitally stable in all dimensions and with non-radial disturbances . Troy demonstrated in 2016 that every conceivable solution to the logarithmic Schrodinger problem has a positive ground condition . In 2017, Hongwei Zhang and Qingying introduced a series of potential wells and tested the logarithmic Schrodinger equation using the fractional logarithmic Sobolev inequality . Standing waves were studied by Jaime and Nataliia using a peak-Gausson profile and a nonlinear logarithmic Schrodinger's equation with interaction.

Bao et al. (2019) investigated two numerical approaches to solving LogSE: a regularized splitting method and a regularized conservative Crank-Nicolson finite difference method (CNFD) . Hongwei et al. looked specifically at how to find an approximation for the log NLS equation problem in unbounded spaces in 2019. Because the logarithmic Schrodinger's equation is not linear and has no boundaries, numerically solving it in unbounded areas is extremely difficult . BAO et al. (2019) developed and discovered an error limit for a regularized finite difference approach applied to the log NLS equation. The emergence of logarithmic nonlinearity in the log NLS equation makes limiting mistakes and developing new numerical approaches more difficult. In 2019, Wazwaz investigated the logarithmic Schrodinger's equation using the variation iteration approach. The research looked into both adding and removing a detuning phrase. That same year, Salman and his colleagues investigated bandpass filters and multiphoton absorption. He also went into greater detail about the average free speed of optical Gaussons that were affected by random events while moving . The Laplace-Adomian decomposition method, according to Gaxiola et al. (2020), could be effective for analyzing visual Gaussons. In 2021, Bianru and Enhua investigated the spectral technique of regularized Lie-Trotter splitting as a means of preventing the regularized space-fractional logarithmic Schrodinger issue from becoming too large . Although the term "detuning" was not generally used, it was included to several numerical models. The method's error analysis was also investigated . Darvishiat et al. published three new logarithmic nonlinear amplitude equations in 2022. The purpose of this research was to identify the source of the Gaussian single waves produced by these logarithmic equations .

Numerical approaches can be used to understand how equations behave in the real world. As a result, our investigation will focus on the linear implicit approach for solving the nonlinear logarithmic Schrodinger's problem. This is the first time that researchers have done something like this.

2.PROBLEM STATEMENT

A numerical solution to the nonlinear logarithmic equation This paper presents Schrodinger's equation using a second-order linear implicit differential technique.

$$i\frac{\partial u}{\partial t}+a\frac{\partial^2 u}{\partial x^2}+b\log\left(|u|^2\right)u=0, \qquad (1)$$

$$i=\sqrt{-1},\ (x_L \leq x \leq x_R,\ 0 \leq t \leq T),$$

What distinguishes a beginning and an end:

$$u(x,0)=g(x),\ x_L \leq x \leq x_R,$$

$$u(x_L,t)=0=u(x_R,t),\ 0 \leq t \leq T,$$

where an is the coefficient of the nonlinear term and b is the coefficient of group velocity spread. We assume that in order to avoid doing a lot of labor,

$$u(x,t)=v(x,t)+iw(x,t), \qquad (2)$$

v and w represent the real functions (x, t). When (1) is changed to (2), the linked system shown below occurs.

$$\frac{\partial v}{\partial t} + a\frac{\partial^2 w}{\partial x^2} + b\log(|u|^2)w = 0, \tag{3}$$

$$\frac{\partial w}{\partial t} - a\frac{\partial^2 v}{\partial x^2} - b\log(|u|^2)v = 0. \tag{4}$$

Exact soliton solution to the log NLS equation is given by [6],[8] ,[17], [26], [29], [30] and [31]

$$u(x,t) = Ae^{-B^2(x-st)^2}e^{i(-kx+\zeta t+\theta)}, \tag{5}$$

where $s = -2ak$, $B = \sqrt{b/2a}$, $\zeta = 2b\log(A) - ak^2 - b$, and $ab > 0$.
The amplitude of Gaussons is denoted by A, while the center of the Gausson's phase is represented by θ. The phase component wave number is ζ, s is the frequency-related Gausson velocity. and B is the inverse width.

In the log NLS equation (1), the three conserved values are [6],[26],[29] and [31]

$$I_1 = \int_{-\infty}^{\infty} |u|^2\, dx, \tag{6}$$

$$I_2 = i\int_{-\infty}^{\infty} [u^* u_x - uu_x^*]dx, \tag{7}$$

$$I_3 = \int_{-\infty}^{\infty} [a\,|u_x|^2 - b\,|u|^2\log|u|^2 + b\,|u|^2]dx, \tag{8}$$

where u^* denote the complex conjugate of u.

3.NUMERICAL METHOD

To develop a numerical scheme for resolving the system denoted by Eqs. (3) and (4), the region $R = (x_L \leq x \leq x_R) \times (t > 0)$ with boundaries defined by the parameters $x = x_L, x_R$ and the axis $t = 0$ is covered by a rectangular mesh of points with coordinates, $x = x_m = x_L + mh$, $m = 0, 1, 2, ..., N$, $t = t_n = nk$, $n = 0, 1, 2, ...,$ $h = \frac{x_R - x_L}{N}$, Where h and k denote the respective space and time increments. We make the assumption that $v(x_m, t_n)$, $w(x_m, t_n)$ are the exact solutions at the point (x_m, t_n), and $V(x_m, t_n)$, $W(x_m, t_n)$ are approximation of the exact solutions. The linearized implicit approach for solving $(3) - (4)$ is provided by

$$\frac{V_m^{n+1} - V_m^{n-1}}{2k} + \frac{a}{h^2}\delta_x^2[\frac{W_m^{n+1} + W_m^{n-1}}{2}] + bz_m^n[\frac{W_m^{n+1} + W_m^{n-1}}{2}] = 0, \tag{9}$$

$$\frac{W_m^{n+1} - W_m^{n-1}}{2k} - \frac{a}{h^2}\delta_x^2[\frac{V_m^{n+1} + V_m^{n-1}}{2}] - bz_m^n[\frac{V_m^{n+1} + V_m^{n-1}}{2}] = 0, \tag{10}$$

where

$$z = \log(|u|^2),$$

Eqs. (9) and (10) may be written as follows:

$$V_m^{n+1} - V_m^{n-1} + \frac{ak}{h^2}\delta_x^2[W_m^{n+1} + W_m^{n-1}] + bkz_m^n[W_m^{n+1} + W_m^{n-1}] = 0, \tag{11}$$

$$W_m^{n+1} - W_m^{n-1} - \frac{ak}{h^2}\delta_x^2[V_m^{n+1} + V_m^{n-1}] - bkz_m^n[V_m^{n+1} + V_m^{n-1}] = 0, \tag{12}$$

by expanding the operators in (11) and (12) we possible get

$$\begin{aligned} & V_m^{n+1} + r_1(W_{m-1}^{n+1} - 2W_m^{n+1} + W_{m+1}^{n+1}) + r_2 z_m^n W_m^{n+1} \\ = \quad & V_m^{n-1} - r_1(W_{m-1}^{n-1} - 2W_m^{n-1} + W_{m+1}^{n-1}) - r_2 z_m^n W_m^{n-1}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} & W_m^{n+1} - r_1(V_{m-1}^{n+1} - 2V_m^{n+1} + V_{m+1}^{n+1}) - r_2 z_m^n V_m^{n+1} \\ = \quad & W_m^{n-1} + r_1(V_{m-1}^{n-1} - 2V_m^{n-1} + V_{m+1}^{n-1}) + r_2 z_m^n V_m^{n-1}, \end{aligned} \tag{14}$$

where

$$r_1 = \frac{ak}{h^2},\ r_2 = bk,\ m = 1, 2, ..., N-1.$$

The system represented by Eqs. (13) and (14) is a three-level scheme that is implicitly linear U^{n+1}, U^n, U^{n-1} in unknown U^{n+1}, this system can be expressed as a matrix vector as follows:

$$M\mathbf{U}^{n+1} = \mathbf{F}(\mathbf{U}^{n-1}, \mathbf{U}^n) \tag{15}$$

Where $\mathrm{U} = [V, W]^t$ and M is a tridiagonal block matrix

$$M(u) = \begin{bmatrix} A_1 & C_1 & 0 & \cdots & 0 \\ B_2 & A_2 & C_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & B_{n-1} & A_{n-1} & C_{n-1} \\ 0 & \cdots & 0 & B_n & A_n \end{bmatrix}.$$

Furthermore, detailed descriptions can be supplied for the following matrix M components:

$$B_m = \begin{bmatrix} 0 & r_1 \\ -r_1 & 0 \end{bmatrix}_m, \; A_m = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}_m, \; C_m = \begin{bmatrix} 0 & r_1 \\ -r_1 & 0 \end{bmatrix}_m,$$

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$$\begin{aligned} a_{12} &= -2r_1 + r_2 z_m^n, \\ a_{21} &= 2r_1 - r_2 z_m^n, \\ m &= 1, 2, \ldots, N. \end{aligned}$$

Using Crout's approach, the solution to the problem described in Equation (15) can be obtained. To begin, we must define two variables. The starting state can be used to calculate U0, which is known at time t = 0. U1 is determined at time k and can represent either the exact solution or any two-level scheme.

4.ACCURACY OF THE SCHEME

To investigate the correctness of the suggested method, we investigate the accuracy of (9), and we substitute the numerical solution V_m^n, W_m^n by the exact solution v_m^n, w_m^n, and the following expansions may be produced by utilizing Taylor's series expansion of all terms in equation (9), round the point (x_m, t_n),

$$\frac{v_m^{n+1} - v_m^{n-1}}{2k} = \frac{\partial v}{\partial t} + \frac{k^2}{6}\frac{\partial^3 v}{\partial t^3} + \ldots,$$

$$\frac{1}{2h^2}\delta_x^2(w_m^{n+1} + w_m^{n-1}) = \frac{\partial^2 w}{\partial x^2} + \frac{h^2}{12}\frac{\partial^4 w}{\partial x^4} + \frac{k^2}{2}\frac{\partial^4 w}{\partial x^2 \partial t^2} + \ldots,$$

$$z_m^n\left(\frac{W_m^{n+1} + W_m^{n-1}}{2}\right) = zW + \frac{k^2}{2} z \frac{\partial^2 W}{\partial t^2} + \ldots, \tag{16}$$

Now, if we replace Eqs. (16) with Eq. (9), we get

$$\begin{aligned} LTE \;=\; & \left[\frac{\partial v}{\partial t} + a\frac{\partial^2 w}{\partial x^2} + bzw\right] + \frac{k^2}{2}\frac{\partial^2}{\partial t^2}\left(\frac{1}{3}\frac{\partial v}{\partial t} + a\frac{\partial^2 w}{\partial x^2}\right. \\ & \left. + bzw\right) + a\frac{h^2}{12}\frac{\partial^4 w}{\partial x^4} + \ldots, \end{aligned} \tag{17}$$

using differential Eq (3) the first bracket is equal to zero and hence

$$LTE = \frac{k^2}{2}\frac{\partial^2}{\partial t^2}\left[\frac{1}{3}\frac{\partial v}{\partial t} + a\frac{\partial^2 w}{\partial x^2} + bzw\right] + a\frac{h^2}{12}\frac{\partial^4 w}{\partial x^4} + \ldots, \tag{18}$$

It has been observed that Equation 18 contains a local truncation error. The approach is considered and implemented at the second-order level. With a similar level of diligence (10), one can verify the plan's accuracy.

Stability of the scheme

Given that van-Neumann stability only applies to linear schemes, the numerical method's stability can be examined by exploiting the scheme's nonlinearity in (1). Achieving linearity in the plan can be performed by quickly removing any term that impedes its progression.

$$i\left(U_m^{n+1} - U_m^{n-1}\right) + r\delta_x^2\left(U_m^{n+1} + U_m^{n-1}\right) + kb\lambda\left(U_m^{n+1} + U_m^{n-1}\right) = 0, \tag{19}$$

Where

$$r = a\frac{k}{h^2}, \quad \lambda = \max(\log|u|^2)$$

To apply the von Neumann stability analysis to Eq. (19), we assume

$$\mathrm{U}_m^n = e^{\alpha nk} e^{i\beta mh}, \quad \delta_x^2 \mathrm{U}_m^n = -4\sin^2(\frac{\beta h}{2})\mathrm{U}_m^n \tag{20}$$

When we plug Eqs. (20) into Eq. (19) we obtain

$$i(e^{\alpha k} - e^{-\alpha k}) - 4r\sin^2(\frac{\beta h}{2})(e^{\alpha k} + e^{-\alpha k}) + kb\lambda(e^{\alpha k} + e^{-\alpha k}) = 0,$$

and it's also possible to write it as

$$e^{\alpha k}(i - \gamma + kb\lambda) = e^{-\alpha k}(i + \gamma - kb\lambda), \tag{21}$$

where

$$\gamma = 4r\sin^2(\frac{\beta h}{2})$$

It's possible to write Eq. (21) like this:

$$e^{2\alpha k} = \frac{[i + (\gamma - kb\lambda)]}{[i - (\gamma - kb\lambda)]} \tag{22}$$

It is simple to see from equation (22), that $|e^{\alpha k}| = 1$, and as a result, we are able to state that the scheme is unconditionally stable in accordance with the von Neumann stability analysis.

Numerical tests

Following this part, a series of tests will be carried out to evaluate the proposed approach's performance, and the findings will be compared to those obtained using the Crank-Nicholson method. Following the addition of the succeeding evaluations, we will now look at the following issues:

Single Soliton

For the sake of applying the precise solution in equation (6), we set the starting condition at $t = 0$, $u(x,0) = Ae^{-B^2x^2}e^{i(-kx+\theta)}$, in the first test, the following criteria are taken into consideration:
$h = 0.1$, $k = 0.0001$, $s = 0.4$, $A = 0.4$, $a = 0.5, b = 1$, $\theta = 0.5$, $x_L = -4$, $x_R = 6$.
In order to investigate the correctness of the suggested technique, we calculate the $L_\infty-$ error norm, which is defined by

$$L_\infty = \|U^n - u^n\|_\infty = \max|U(x_m, t_m) - u(x_m, t_m)|$$

In the second test, the following criteria are taken into consideration:

$$\begin{aligned} h &= 0.05,\ k = 0.001,\ s = 0.4,\ A = 0.4,\ a = 0.5, \\ b &= 1,\ \theta = 0.5,\ x_L = -4,\ x_R = 8 \end{aligned}$$

Table 1 shows a comparison of the error analysis of the linearized implicit scheme and the Crank-Nicolson technique using L norms. Table 2 shows the retained quantities of a solitary soliton as determined by the Crank-Nicolson method and the linearized implicit scheme. When the same parameters are applied to constant soliton values, the two tables show that the linear implicit scheme and the Crank-Nicolson approach produce almost identical valid solutions. As shown in Table 3, as h lowers, the linearized implicit gets more precise. The linearized implicit technique was used to find a single soliton solution with the parameters h=0.1, k=0.0001, and s=0.4, as illustrated in Figure 1. As shown in Figure 2, the linearized implicit technique found a single soliton solution with the parameters h=0.05, k=0.001, and s=0.4.

Table 1. First test: (a) error analysis; compare the linearized implicit scheme and the Crank-Nicolson

method

$h = 0.1,\ k = 0.0001$				
time	$L - M$		$C - N$	
	$L_\infty(V)$	$L_\infty(W)$	$L_\infty(V)$	$L_\infty(W)$
0.00	0.000000	0.000000	0.000000	0.000000
1.00	0.000833	0.001875	0.000833	0.001875
2.00	0.001345	0.001596	0.001344	0.001595
3.00	0.002503	0.003055	0.002503	0.003054
4.00	0.002845	0.003668	0.002844	0.003667
5.00	0.004200	0.003933	0.004200	0.003933

Table 2. First test: (b) quantities preserved compare of the Linearized implicit scheme and Crank Nicolson method

$h = 0.1,\ k = 0.0001$						
time	$L - M$			$C - N$		
	I_1	I_2	I_3	I_1	I_2	I_3
0.00	0.200530	-0.159582	0.783269	0.200530	-0.159582	0.783269
1.00	0.200530	-0.159572	0.783249	0.200530	-0.159571	0.783248
2.00	0.200530	-0.159579	0.783265	0.200530	-0.159579	0.783266
3.00	0.200530	-0.159573	0.783251	0.200530	-0.159572	0.783251
4.00	0.200530	-0.159577	0.783259	0.200530	-0.159578	0.783260
5.00	0.200530	-0.159577	0.783257	0.200530	-0.159577	0.783257

Table 3. Second test error Analysis and quantities Preserved using Linearisation method

$h = 0.05,\ k = 0.001$					
time	$L_\infty(V)$	$L_\infty(W)$	I_1	I_2	I_3
0.00	0.000000	0.000000	0.200530	-0.160213	0.784259
2.00	0.000342	0.000402	0.200530	-0.160213	0.784258
4.00	0.000722	0.000929	0.200530	-0.160213	0.784258
6.00	0.001286	0.000984	0.200530	-0.160213	0.784258
8.00	0.001822	0.001498	0.200530	-0.160213	0.784257
10.00	0.001739	0.002074	0.200530	-0.160213	0.784257

Collision of two solitons

A computer experiment then investigates how two solitons can make bidirectional contact. This can be performed by using the following document.

$$u(x,t) = A\sum_{m=1}^{q} \exp[-B^2(x - x_m - s_m t)^2]\exp(i(-k_m(x - x_m) + \zeta_m t + \theta)),$$

$$s_m = -2ak_m,\ B = \sqrt{b/2a}, \zeta_m = 2b\log(A) - ak_m^2 - b,\ q = 2 \ and \ ab > 0,$$

During the third examination, the following elements are taken into account:

$$h = 0.1,\ k = 0.0001,\ s_1 = 0.8,\ s_2 = -0.4,\ A = 0.4,\ a = 0.5,$$

$$b = 1,\ \theta = 0.5,\ x_1 = 0,\ x_2 = 4.5,\ x_L = -4,\ x_R = 10$$

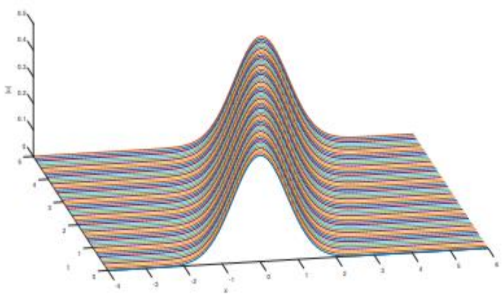

Fig. 1. To mimic a solitary soliton, we used an implicit linearization technique. The given

parameters were h = 0.1, k = 0.0001, s = 0.4, 0 t 5, and 0 t 5.

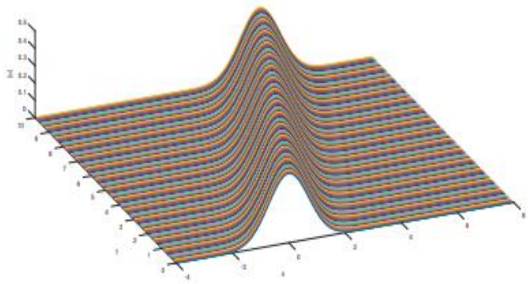

Fig.2. To simulate a solitary soliton, a linearization implicit scheme was used with h = 0.05, k = 0.001, s = 0.4, and t = 0 to 10.

Table 4. The final measure of assessment is a comparison of the quantities retained by the Linearized implicit plan and the Crank Nicolson technique for the interaction of two solitons.

$h = 0.1,\ k = 0.0001$						
time	$L-M$			$C-N$		
	I_1	I_2	I_3	I_1	I_2	I_3
0.00	0.401069	-0.159233	1.613564	0.401069	-0.159233	1.613564
1.00	0.401069	-0.159125	1.613448	0.401069	-0.159125	1.613450
2.00	0.401068	-0.159250	1.613313	0.401069	-0.158250	1.613315
3.00	0.401069	-0.157636	1.610116	0.401068	-0.157637	1.610117
4.00	0.401069	-0.158954	1.613604	0.401068	-0.158945	1.613603
5.00	0.401069	-0.158669	1.611677	0.401068	-0.158667	1.611674

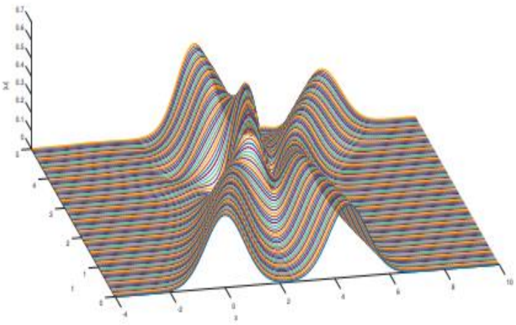

Fig. 3. A linearization implicit method collision happens between two solitons at time t = 0. Their values are h = 0.1, k = 0.0001, x1 = 0, and x2 = 4.5.

Table 3 shows a side-by-side comparison of the linearized implicit method and the Crank-Nicholson approach for a collision involving two solitons moving in opposite directions at varying velocities from left to right. -The two accessible options are 0.4 and 0.8. The data in Figure 3 clearly show that the two solitons recover to their initial form following the collision.

Interaction of Three Solitons

This can be performed by using the following document.

$$u(x,t) = A\sum_{m=1}^{q} \exp\left[-B^2\left(x - x_m - s_m t\right)^2\right]\exp\left(i\left(-k_m\left(x - x_m\right) + \zeta_m t + \theta\right)\right),$$

The fourth evaluation looks at the following aspects.

$$\begin{aligned} h &= 0.1,\ k = 0.0001,\ s_1 = 2.4 = -s_3,\ s_2 = 0.1,\ q = 3,\ A = 0.4,\ a = 0.5, \\ b &= 1,\ \theta = 0.5,\ x_1 = -7,\ x_2 = 0,\ x_3 = 7,\ x_L = -15,\ x_R = 15 \end{aligned}$$

Table 5. Fourth test conserved quantities of three solitons interaction by Linearization implicit scheme

$h = 0.1$, $k = 0.0001$			
time	I_1	I_2	I_3
0.00	0.601591	-0.039906	3.414252
1.00	0.601591	-0.039904	3.412848
2.00	0.601592	-0.040613	3.410579
3.00	0.601592	-0.040668	3.397667
4.00	0.601592	-0.043239	3.411051
5.00	0.601592	-0.042279	3.413648

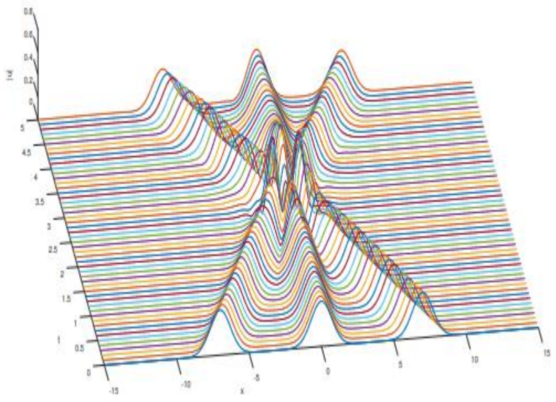

Fig. 4. Three solitons with the following properties collide at speeds ranging from 0 to 5 times the speed of light: h = 0.1, k = 0.0001, x1 = -7, x2 = 0, and x3 = 7. Implicit linearization is proof for this.

Figure 5 depicts a computer demonstration of the linearized implicit system in action. This method governs the collision of three slithons going in opposite directions who turn left. There is a 0.1 mile per hour difference between -2.4 and 2.4 miles per hour. Figure 4 shows that the solitones restore to their initial arrangement after the impact.

5.CONCLUSION

We investigated the log NLS problem theoretically using linearization in this study. The linearization strategy produced findings that were nearly as exact as the Crank-Nicolson approach. Furthermore, values that were kept were kept. In the absence of restrictions, the linearization approach has comparable stability, temporal and spatial accuracy, and accuracy to the second order as the Crank-Nicolson method. It also shows how three or two solitons might come into bidirectional contact. Even after being struck by two to three soltons, the ductility remained unaltered. Data Access Capability The research is supported by the study statistics offered in the publication. Confrontational inconsistencies that appear to be true The study's authors claim that they have no obvious conflicts of interest in this investigation.

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